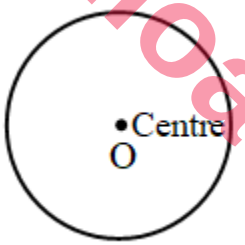
Course: General Mathematics and Statistics (6401)

Semester: Autumn, 2021

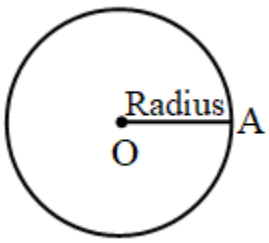
ASSIGNMENT No. 2

Q.1 Write down construction of different types of circles (At least 4) along with their diagrams.

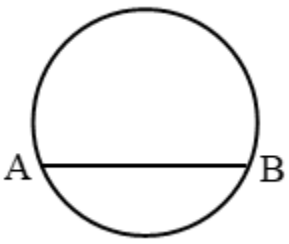
1. **Circle:** A circle is a collection of all those points in a plane that are at a given constant distance from a given fixed point in the plane.
2. **Centre:** Circle is a closed figure made up of points in a plane that are at the same distance from a fixed point, called the centre of the circle. In the figure O is the centre.



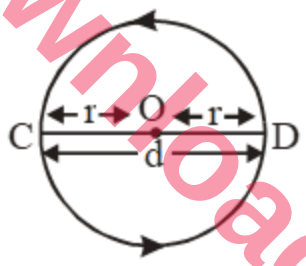
1. **Radius:** The constant distance from its centre is called the radius of the circle. In the figure, OA is radius



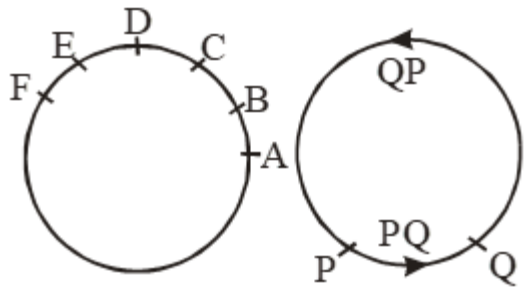
2. **Chord:** A line segment joining two points on a circle is called a chord of the circle. In the figure, AB is a chord of the circle. If a chord passes through centre then it is longest chord. PQ, PR, and ST are chords of the circle. Chord ST passes through the centre, hence it is a diameter.



3. **Diameter:** A chord passing through the centre of a circle is called the diameter of the circle. A circle has an infinite number of diameters. CD is the diameter of the circle as shown in the figure. If d is the diameter of the circle then d = 2r. where r is the radius. or the longest chord is called diameter. In the figure, AB is the diameter and the arcs CD and DC are semicircles.



4. **Arc:** A continuous piece of a circle is called an arc. Let A,B,C,D,E,F be the points on the circle. The circle is divided into different pieces. Then, the pieces AB, BC, CD, DE, EF etc. are all arcs of the circle.



Let P,Q be two points on the circle. These P, Q divide the

circle into two parts. Each part is an arc. These arcs are denoted in anti-clockwise direction

from P to Q as $\overset{\frown}{PQ}$ and form Q to P as $\overset{\frown}{QP}$.

The counter clockwise direction distinguishes between these two arcs $\overset{\frown}{PQ}$ and $\overset{\frown}{QP}$.

The length of arc $\overset{\frown}{PQ}$ can be less than, equal to or greater than the length of the arc $\overset{\frown}{QP}$

i.e., (i) $\ell(\overset{\frown}{PQ}) < \ell(\overset{\frown}{QP})$ (ii) $\ell(\overset{\frown}{PQ}) = \ell(\overset{\frown}{QP})$

(iii) $\ell(\overset{\frown}{PQ}) > \ell(\overset{\frown}{QP})$

when $\ell(\overset{\frown}{PQ}) < \ell(\overset{\frown}{QP})$, then the arc $(\overset{\frown}{PQ})$ is called a minor arc.

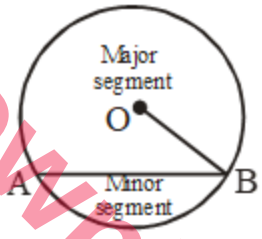
If $\ell(\overset{\frown}{PQ}) = \ell(\overset{\frown}{QP})$, then the arc $\overset{\frown}{PQ}$ and $\overset{\frown}{QP}$ are called semi circle.

At this time points of arc at end of diameter.

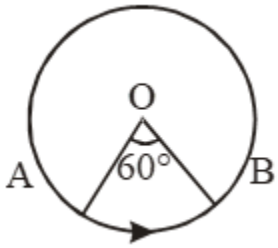
And when $\ell(\overset{\frown}{PQ}) > \ell(\overset{\frown}{QP})$, then the arc $\overset{\frown}{PQ}$ is called a major arc.

5. **Circumference of a circle:** The perimeter of a circle is called its circumference. The circumference of a circle of radius r is $2\pi r$.
6. **Semicircle:** The diameter of a circle divides the circle into two equal parts. Each part is called a semi-circle. We can also say that half of a circle is called a semi¬circle. In the figure, AXB and AYB represents two semi-circles.
7. **Segment:** Let AB be a chord of the circle. Then, AB divides the region enclosed by the circle (i.e., the circular disc) into two parts. Each of the parts is called a segment of the circle. The segment, containing the minor arc is called minor segment and the segment, containing the major arc, is called the major

segment and segment of a circle is the region between an arc and chord of the circle.



8. **Central Angles:** Consider a circle. The angle subtended by an arc at the centre O is called the central angle. The vertex of the central angle is always at the centre O.



9. **Degree measure of an arc:** Degree measure of a minor arc is the measure of the central angle subtended by the arc.

In the figure, the measure of the arc $\overset{\frown}{PQ}$ is 60°

i.e., $m\overset{\frown}{PQ} = 60°$.

The measure of a major arc is $360° - m\overset{\frown}{PQ}$ the degree measure of the corresponding minor arc.

The degree measure of the major arc is $360° - 60° = 300°$

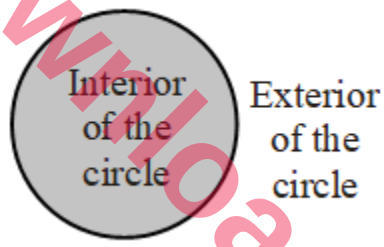
$\therefore\ m\overset{\frown}{QP} = 300°$.

The degree measure of the circumference of the circle is always 360°.

10. **Interior and Exterior of Circle**

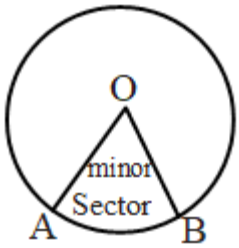
A circle divides the plane on which lies into three parts.

(i) Inside the circle. which is called the interior of the circle
(ii) Circle
(iii) Outside the circle, which is called the exterior of the circle.
The circle and its interior make up the circular region.



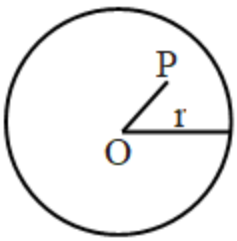
11. **Sector:**

A sector is that region of a circular disc which lies between an arc and the two radii joining the extremities of the arc and the centre. OAB is a sector as shown in the figure. Quadrant: One fourth of a circular disc is called a quadrant.



12. **Position of a point:**

Point Inside the circle: A point P, such that OP < r, is said to lie inside the circle.



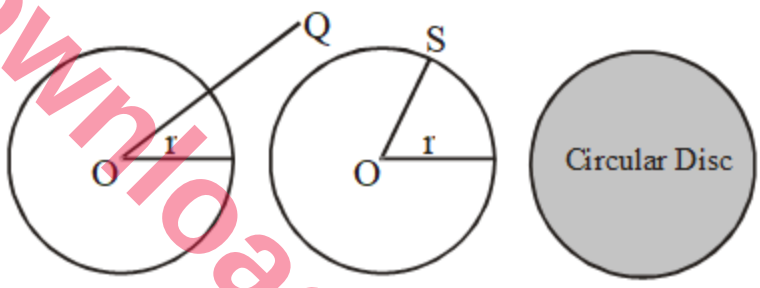
The point inside the circle is also called **interior point**. (Example : Centre of cirle)

Point outside the circle: A point Q, such that OQ > r, is said to lie outside the circle C (O, r) = {X, OX = r}

The point outside the circle is also called **exterior point**.

Point on the circle: A point S, such that OS = r is said to lie on the circle C(O, r) = {X ,OX = r}.

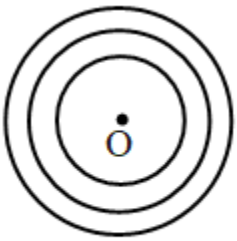
Circular Disc: It is defined as a set of interior points and points on the circle. In set notation, it is written as : C(O, r) = {X : P OX ≤ r}



13. **Concentric Circles:**

Circles having the same centre and different radius are said to be concentric circles.

Remark. The word 'radius' is used for a line segment joining the centre to any point on the circle and also for its length.

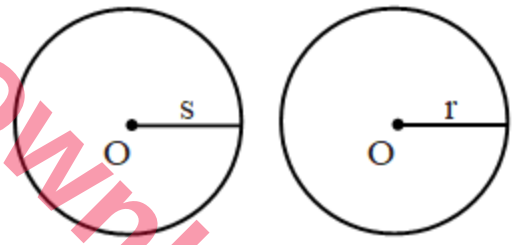


Concentric Circles

14. **Congruence of Circles & Arcs**

Congruent circles: Two circles are said to be congruent if and only if, one of them can be superposed on the other, so as the cover it exactly. It means two circles are congruent if and only if, their radii are equal.

i.e., C (O, r) and C (O' , r) are congruent if only if r = s.



Congruent circles if r = s

15. **Congruent arcs:** Two arcs of a circle are congruent, if either of them can be superposed on the other, so as to cover it exactly. It is only possible, if degree measure of two arcs are the same.

Q.2 Prove the following identities:

i. (secθ + tanθ) (1- sinθ) = cosθ

(sec θ + tan θ) (1 – sin θ)

= (1/cos θ + sin θ/cos θ)(1−sin θ)

= {(1+sin θ)/cos θ}(1 − sin θ)

= (1− sin θ)(1 + sin θ)/cos θ

$(a + b)(a – b)= a^2 – b^2$, So,

$(1 + \sin \theta)(1 – \sin A) = 1 – \sin^2\theta$.

$= (1 – \sin^2 \theta)/\cos \theta$

$\sin^2\theta + \cos^2\theta = 1$. So,

$1 – \sin^2\theta = \cos^2\theta$

Putting it in above expression,

$= \cos^2\theta/\cos \theta$

= cos θ

ii. $\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$

$$\frac{\sec^2 x}{\sec^2 x - 1}$$

$$= \frac{\sec^2 x}{\tan^2 x}$$

$$= \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{1}{\cos^2 x} \times \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x}$$

$$= \csc^2 x$$

Q.3 Define volume. Show how to find the volume of different shapes with diagrams. Also differentiate between area and volume.

The area is defined as the region covered by two-dimensional shapes. The area of different shapes depends on their dimensions. It is measured in square units.

The 2d shapes include circle, triangle, square, rectangle, parallelogram, pentagon, hexagon, etc. Hence, all these shapes have different areas.

Volume is a parameter that is defined only for three-dimensional objects. For each three-dimensional shape, such as sphere, cube, cuboid, cylinder, cone, etc., the volume is different. It is measured in cubic units.

Area	**Volume**
It is to be noted that area is always defined for two dimensions objects or plane figures	The volume is always defined for a three-dimensional object.
The area is the amount of space occupied by a two-dimensional flat object in a plane.	Volume is defined as the space occupied by a three-dimensional object.
The unit of area is in square units.	The unit of volume is in cubic units.

It is measured in 2 dimension	It is measured in 3 dimension
Examples: Square, circle, rectangle, etc.	Examples: sphere, cube, cylinder, etc.

Difference of Area and Volume of Shapes

Let us see the areas of different shapes here:

Name of the shape	**Area**
Circle	Πr^2 (r = radius)
Semicircle	½ πr^2 (r = radius)
Triangle	½ × base × height
Square	$Side^2$
Rectangle	Length × Width
Parallelogram	Base × Height
Rhombus	Side × Height
Trapezoid	½ (Sum of parallel side) × height

Let us find the volumes of different shapes here:

Name of the shape	**Volume**
Cube	3 Side
Cuboid	Length × Width × Height
Sphere	$(4/3)\pi r^3$
Hemisphere	⅔ (πr^3)
Cylinder	$\pi r^2 h$
Cone	$1/3(\pi r^2 h)$
Prism	Area of Base ×

	Height

Q.4 Write a detailed note on classification and tabulation of data and explain their types. Draw graphs where needed.

The primary difference between classification and tabulation is that the process of classifying data int groups is known as classification of data, whereas tabulation is the act of presenting data in tabular form, for better interpretation.

After the collection of data is completed, it is prepared for analysis. As the data is raw, it needs to be transformed in such a way, that it is appropriate for analysis. The form of data, highly influences the result of analysis and so, to get positive results, the data preparation should be proper. There are various steps of data preparation, which include editing, coding, classification, tabulation, graphical representation and so on.

For a layperson, classification and tabulation are same, but the fact is they are different, as the former is a means to sort data, for further analysis while the latter is used to present data.

BASIS FOR COMPARISON	**CLASSIFICATION**	**TABULATION**
Meaning	Classification is the process of grouping data into different categories, on the basis of nature, behavior, or common characteristics.	Tabulation is a process of summarizing data and presenting it in a compact form, by putting data into statistical table.
Order	After data collection	After classification
Arrangement	Attributes and variables	Columns and rows
Purpose	To analyse data	To present data
Bifurcates data into	Categories and sub-categories	Headings and sub-headings

Classification refers to a process, wherein data is arranged based on the characteristic under consideration, into classes, or groups, as per resemblance of observations. Classification puts the data in a condensed form, as it removes unnecessary details that helps to easily comprehend data.

The data collected for the first time is raw data and so it is arranged in haphazard manner, which does not provide a clear picture. The classification of data reduces the large volume of raw data into homogeneous

groups, i.e. data having common characteristics or nature are placed in one group and thus, the whole data is bifurcated into a number of groups. there are four types of classification:

- Qualitative Classification or Ordinal Classification
- Quantitative Classification
- Chronological or Temporal Classification
- Geographical or Spatial Classification

Tabulation refers to a logical data presentation, wherein raw data is summarized and displayed in a compact form, i.e. in statistical tables. In other words, it is a systematic arrangement of data in columns and rows, that represents data in concise and attractive way. One should follow the given guidelines for tabulation.

- A serial number should be allotted to the table, in addition to the self explanatory title.
- The statistical table is required to be divided into four parts, i.e. Box head, Stub, Caption and Body. The complete upper part of the table that contains columns and sub-columns, along with caption, is the Box Head. The left part of the table, giving description of rows is called stub. The part of table that contains numerical figures and other content is its body.
- Length and Width of the table should be perfectly balanced.
- Presentation of data should be such that it takes less time and labor to make comparison between various figures.
- Footnotes, explaining the source of data or any other thing, are to be presented at the bottom of the table.

The paramount differences between classification and tabulation are discussed in the points given below:

1. The process of arranging data into different categories, on the basis of nature, behavior, or common characteristics is called classification. A process of condensing data and presenting it in a compact form, by putting data into statistical table, is called tabulation.
2. Classification of data is done after data collection process is completed. On the other hand, tabulation follows classification.
3. Data classification is based on similar attributes and variables of the observations. Conversely, in tabulation the data is arranged in rows and columns, in a systematic way.
4. Classification of data is performed with the objective of analysing data in order to draw inferences. Unlike tabulation, which aims at presenting data, to ensure easy comparison of various figures.
5. In classification, data is bifurcated into categories and sub-categories while in tabulation data is divided into headings and sub-headings.

Q. 5 What is variance? Differentiate between variance and standard deviation. Write down their properties and application in field of science.

Standard deviation and variance are basic mathematical concepts that play important roles throughout the financial sector, including the areas of accounting, economics, and investing. In the latter, for example, a firm

grasp of the calculation and interpretation of these two measurements is crucial to the creation of an effective trading strategy.

Standard deviation and variance are both determined by using the mean of a group of numbers in question. The mean is the average of a group of numbers, and the variance measures the average degree to which each number is different from the mean. The extent of the variance correlates to the size of the overall range of numbers—meaning the variance is greater when there is a wider range of numbers in the group, and the variance is less when there is a narrower range of numbers.

- Standard deviation looks at how spread out a group of numbers is from the mean, by looking at the square root of the variance.
- The variance measures the average degree to which each point differs from the mean—the average of all data points.
- The two concepts are useful and significant for traders, who use them to measure market volatility.

Standard Deviation

Standard deviation is a statistic that looks at how far from the mean a group of numbers is, by using the square root of the variance. The calculation of variance uses squares because it weighs outliers more heavily than data closer to the mean. This calculation also prevents differences above the mean from canceling out those below, which would result in a variance of zero.

Standard deviation is calculated as the square root of variance by figuring out the variation between each data point relative to the mean. If the points are further from the mean, there is a higher deviation within the date; if they are closer to the mean, there is a lower deviation. So the more spread out the group of numbers are, the higher the standard deviation.

Variance

The variance is the average of the squared differences from the mean. To figure out the variance, first calculate the difference between each point and the mean; then, square and average the results.

For example, if a group of numbers ranges from 1 to 10, it will have a mean of 5.5. If you square the differences between each number and the mean, and then find their sum, the result is 82.5. To figure out the variance, divide the sum, 82.5, by N-1, which is the sample size (in this case 10) minus 1. The result is a variance of 82.5/9 = 9.17. Standard deviation is the square root of the variance so that the standard deviation would be about 3.03.

Because of this squaring, the variance is no longer in the same unit of measurement as the original data. Taking the root of the variance means the standard deviation is restored to the original unit of measure and therefore much easier to interpret.

For traders and analysts, these two concepts are of paramount importance as they are used to measure security and market volatility, which in turn plays a large role in creating a profitable trading strategy.

Standard deviation is one of the key methods that analysts, portfolio managers, and advisors use to determine risk. When the group of numbers is closer to the mean, the investment is less risky; when the group of numbers is further from the mean, the investment is of greater risk to a potential purchaser.

Securities that are close to their means are seen as less risky, as they are more likely to continue behaving as such. Securities with large trading ranges that tend to spike or change direction are riskier. In investing, risk in itself is not a bad thing, as the riskier the security, the greater potential for a payout.