Course: Mathematics III (6448)
Semester: Autumn, 2021

ASSIGNMENT No. 1

Q.1 Discuss the standard form of an ellipse.

An **ellipse** is the locus of a point whose sum of the distances from two fixed points is a constant value. The two fixed points are called the foci of the ellipse, and the equation of the ellipse is x2 / a2+y2 / b2=1. Here a is called the semi-major axis and b is called the semi-minor axis of the ellipse.

There are a lot of ellipses besides the ones in the standard form. If you take any quadratic equation of the form

Ax2+Bxy+Cy2+Dx+Ey+F=0

where A,B,C,D,E,A,B,C,D,E, and FF are constants, the discriminant B2−4AC is negative, then the solutions form an ellipse. (There's also a condition on FF to make sure that there are any solutions at all.) But that ellipse needn't be centered on the origin and its axes needn't be aligned horizontally and vertically.

The different properties of an ellipse are as given below,

- An ellipse is created by a plane intersecting a cone at the angle of its base.
- All ellipses have two foci, a center, and a major and minor axis.
- The sum of the distances from any point on the ellipse to the two foci gives a constant value.
- The value of eccentricity for all ellipses is less than one.

The equation of the ellipse can be derived from the basic definition of the ellipse: An ellipse is the locus of a point whose sum of the distances from two fixed points is a constant value. Let the fixed point be P(x, y), the foci are F and F'. Then the condition is PF + PF' = 2a. This on further substitutions and simplification we have the equation of the ellipse as x2 / a2+y2 / b2=1.

The eccentricity of the ellipse refers to the measure of the curved feature of the ellipse. For an ellipse, the eccentric is always greater than one. (e < 1). Eccentricity is the ratio of the distance of the focus and one end of the ellipse, from the center of the ellipse. If the distance of the focus from the center of the ellipse is 'c' and the distance of the end of the ellipse from the center is 'a', then eccentricity e = c/a.

The ellipse has two foci, F and F'. The midpoint of the two foci of the ellipse is the center of the ellipse. All the measurements of the ellipse are with reference to these two foci of the ellipse. As per the definition of an ellipse, an ellipse includes all the points whose sum of the distances from the two foci is a constant value.

The line passing through the two foci and the center of the ellipse is called the transverse axis of the ellipse. The major axis of the ellipse falls on the transverse axis of the ellipse. For an ellipse having the center and the foci on the x-axis, the transverse axis is the x-axis of the coordinate system.

Q.2 a) Discuss the rose curves.

In mathematics, a **rose** or **rhodonea curve** is a sinusoid specified by either the cosine or sine functions with no phase angle that is plotted in polar coordinates. Rose curves or "rhodonea" were named by the Italian mathematician who studied them, Guido Grandi, between the years 1723 and 1728.

Having seen that there were more than 1 K viewers in a day, I now add more.

The 2-D polar coordinates P(r,θ), r = √x2+y2≥0. It represents length of the position vector <r,θ>.θ determines the direction. It increases for anticlockwise motion of P about the pole O. For clockwise rotation, it decreases. Unlike r, θ admit negative values.

r-negative tabular values can be used by artists only.

The polar equation of a rose curve is either r=acosnθ.

n is at your choice. Integer values 2,, 3, 4.. are preferred for easy counting of the number of petals, in a period. n = 1 gives 1-petal circle.

To be called a rose, n has to be sufficiently large and integer + a fraction, for images looking like a rose. For integer values, the petals might be redrawn, when the drawing is repeated over successive periods. The period of both sinnθandcosnθ is 2πn.

The number of petals for the period [0,2πn] will be n or 2n (including r-negative n petals) according as n is odd or even, for 0≤θ≤2π. Of course, I maintain that r is length ≥0, and so non-negative. For Quantum Physicists, r > 0.

Foe example, consider r=2sin3θ. The period is 2π3 and the number of petals will be 3. In continuous drawing. r-positive and r-negative petals are drawn alternately. When n is odd, r-negative petals are same as r-positive ones. So, the total count here is 3.

Prepare a table for (r,θ), in one period [0,2π3], for θ=0,π12,2π12,3π12,...8π12. Join the points by smooth curves, befittingly. You get one petal. You ought to get the three petals for 0≤θ≤2π..

For r=cos3θ, the petals rotate through half-petal angle = π6, in the clockwise sense.

A sample graph is made for r=4cos6θ, using the Cartesian equivalent. It is r-positive 6-petal rose, for 0≤θ≤2π.

graph{(x^2+y^2)^3.5-4(x^6-15x^2y^2(x^2-y^2)-y^6)=0}.

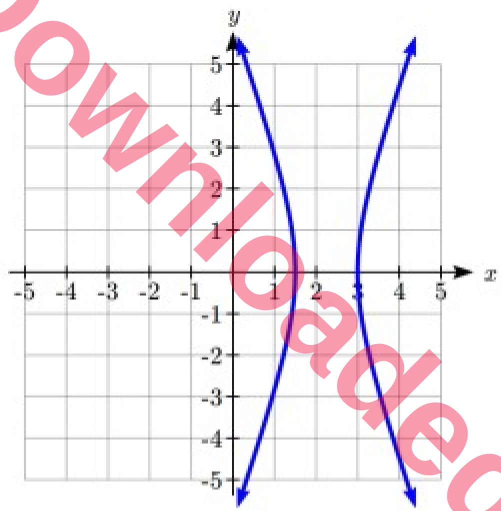
b) Find the polar form of the conic given a focus at the origin, e=3 and directrix y= –2.

Most of us are familiar with orbital motion, such as the motion of a planet around the sun or an electron around an atomic nucleus. Within the planetary system, orbits of planets, asteroids, and comets around a larger celestial body are often elliptical. Comets, however, may take on a parabolic or hyperbolic orbit instead. And, in reality, the characteristics of the planets' orbits may vary over time. Each orbit is tied to the location of the celestial body being orbited and the distance and direction of the planet or other object from that body. As a result, we tend to use polar coordinates to represent these orbits.

In an elliptical orbit, the **periapsis** is the point at which the two objects are closest, and the **apoapsis** is the point at which they are farthest apart. Generally, the velocity of the orbiting body tends to increase as it approaches the periapsis and decrease as it approaches the apoapsis. Some objects reach an escape velocity, which results in an infinite orbit. These bodies exhibit either a parabolic or a hyperbolic orbit about a body; the orbiting body breaks free of the celestial body's gravitational pull and fires off into space. Each of these orbits can be modeled by a conic section in the polar coordinate system.

We are given e = 3 and p = 2. Since the directrix is vertical and at a positive x value, we use the equation involving cos with the positive sign.

$$r = \frac{(3)(2)}{1+3\cos(\theta)} = \frac{6}{1+3\cos(\theta)}$$



Q.3 a) Find the asymptotes of the curve. R = a tanθ

Given r = a tanθ, i.e. 1/r = cos θ/a sin θ ......(1)

Let $u = \frac{1}{r}$ then $u \to 0$ implies $\frac{\cos\theta}{a\sin\theta} \to 0,$

i.e. $\cos\theta \to 0$ impling $\theta \to (2n+1)\frac{\pi}{2}$

Now $$\frac{du}{d\theta} = \frac{1}{a}(-\text{cosec}^2\theta) = -\frac{1}{a\sin^2\theta}$$

$$p = \underset{\theta\to\theta_1}{\text{Lt}}\left(-\frac{d\theta}{du}\right) = -\underset{\theta\to\theta_1}{\text{Lt}}\left(-a\sin^2\theta\right) = a(\sin\theta_1)^2$$

$$= a[\sin(2n+1)\pi/2]^2 = a(-1)^{2n} = a \quad .....(3)$$

By definition,

$$p = r\sin(\theta_1 - \theta) \quad ....(4)$$

From (3) and (4)

a = r sin((2nπ + π)/2 – θ))

a = r sin(nπ + π/2 – θ) = r(–1)^{n}sin(π)/2 – θ)

a = ±r cos θ or r cosθ = ±a ......(5)

b) Find the relative extreme value of, $f(x) = 4x^4 - 8x^3 + 22x^2 - 24x + 1$

f (x) = $4x^4 - 8x^3 + 22x^2 - 24x + 1$.

f ¢ (x) = $16x^3 - 24x^2 + 44x - 24 = 4(4x^3 - 6x^2 + 11x - 6)$

f ¢ (x) = $4(2x - 1)(2x^2 - 5x + 6)$

For f (x) to be increasing, we must have

f ¢ (x) > 0

Þ $4(x - 1)(x^2 - 5x + 6) > 0$

Þ $(x - 1)(x^2 - 5x + 6) > 0$

Þ $(x - 1)(x - 2)(x - 3) > 0$ [∵ 4 > 0]

Þ 1 < x < 2 or 3 < x < ¥

Þ x Î (1, 2) È (3, ¥)

So, f (x) is increasing on (1, 2) È (3, ¥)

For f (x) to be decreasing, we must have

f ¢ (x) < 0

Þ $4(x - 1)(x^2 - 5x + 6) > 0$

Þ $(x - 1)(x^2 - 5x + 6) < 0$ [∵ 4 > 0]

Þ $(x - 1)(x - 2)(x - 3) < 0$

Þ 2 < x < 3 or x < 1

Þ x Î (2, 3) È (–¥, 1)

So, f (x) is decreasing on (2, 3) È (–¥, 1)

Q.4 a) Find the distance between points: (1, 0, 2) and (2, 4, -3).

$$d = \sqrt{(2-1)^2 + (4-0)^2 + (-3-2)^2}$$

$$d = \sqrt{(1)^2 + (4)^2 + (-5)^2}$$

$$d = \sqrt{1 + 16 + 25}$$

$$d = \sqrt{42}$$

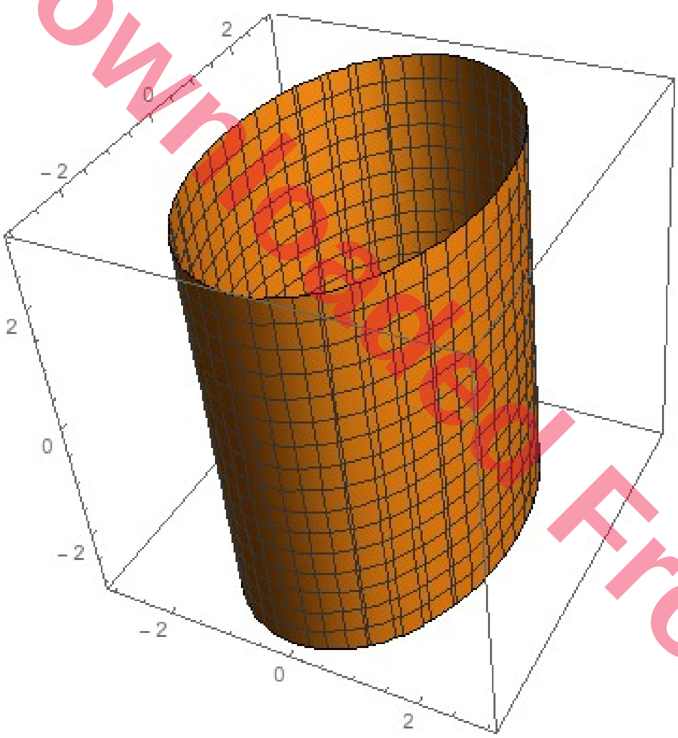
$$d = 6.480741$$

b) Determine whether the points P(2,-1,4), Q(5,4,6) and R(-4,-11,0) are collinear.

Collinear test:	The points are collinear (they located on the same line).		
Determinant of the points:	0		
Rank of the determinant:	2		
Parametric lines equation:	x = 2 + 3t	y = -1 + 5t	z = 4 + 2t

Vectors: n_1, n_2, n_3,	3i + 5j + 2k	-6i – 10j – 4k	-9i – 15j – 6k
Area between the points:	0		
Plane equation:	--		

Q.5 a) Sketch the cylinder: $\frac{x^2}{9} + \frac{y^2}{4} = 1$



b) Classify and sketch the surface $4x^2 - 3y^2 + 12z^2 + 12 = 0$.

Begin by writing the equation in standard form.

$$4x^2 - 3y^2 + 12z^2 + 12 = 0$$ Write original equation.

$$\frac{x^2}{-3} + \frac{y^2}{4} - z^2 - 1 = 0$$ Divide by -12.

$$\frac{y^2}{4} - \frac{x^2}{3} - \frac{z^2}{1} = 1$$ Standard form

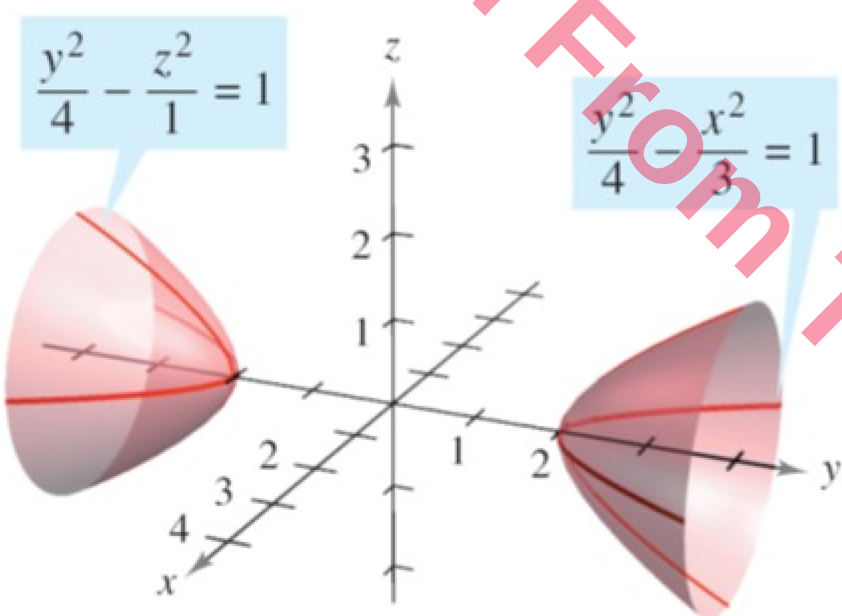
You can conclude that the surface is a hyperboloid of two sheets with the *y*-axis as its axis.

To sketch the graph of this surface, it helps to find the traces in the coordinate planes.

xy-trace ($z = 0$): $\frac{y^2}{4} - \frac{x^2}{3} = 1$ Hyperbola

xz-trace ($y = 0$): $\frac{x^2}{3} + \frac{z^2}{1} = -1$ No trace

yz-trace ($x = 0$): $\frac{y^2}{4} - \frac{z^2}{1} = 1$ Hyperbola



Hyperboloid of two sheets:
$\frac{y^2}{4} - \frac{x^2}{3} - z^2 = 1$